

Modeling of Emergence

Mostafa Zahri

Taibah University
Al Madinah Al Munawwarah
07/02/1433

Different Forms of Emergence: Interaction Individuals

Cells



Bees



Birds



- Self-Organization,
- Emergence,
- Non-linear systems,
- Distributive networks,
- Collective intelligence,
- Swarm intelligence,
- From individual to collective intelligence ?

Public market in Marakech



Overview

- Living in Large Groups
- Complexity and Rules of Emergence
- Modeling of Emergence
- Energy-based-model (Zahri 2009)
- Finite metric space
- Continuous metric space
- Convergence Theorems & Simulation
- Very Known Self-Organizing Dynamics

Living in Large Groups







- A large group offers protection from predators.
- Large groups can perform organised evasive manoeuvres to outwit predators.
- Swimming in a school of fishes also can result in significant energy savings, due to hydrodynamical advantages of streamlining.

What is this all about?

- "An individual ant is not very bright, but ants in a colony, operating as a **collective**, do remarkable things.

- A single neuron in the human brain can respond only to what the neurons connected to it are doing, but all of them **together** can be Albert Einstein."

Deborah Gordon (Stanford University)

- **Our main concern is to observe systems where simple agents (individuals, units, group members) together behave in complicated ways.**

Complexity of Emergent Population

Complex systems have behavioral and structural features in common:

- Complex systems are **non-linear** Relationships
- These Relationships contain **feedback loops**
- Complex systems are **time dependent**: *they change over time, and prior states may have an influence on present states.*
- Complex systems may be **nested**: *The components of a complex system may themselves be complex systems. (cell-organism-colony-ecosystem)*
- May produce **emergent phenomena**.

What are the rules of the Emergence !

- Agents **follow simple rules** generate complex structures.
- Agents do **not follow orders from a leader**.
- Agents **behave locally** and adapt their position to the other members of the group.
- Swarm Intelligence: **artificial intelligence technique** based on the study of collective behavior in self-organized systems.

Modeling of Emergence

Novel Type of Phase Transition in a System of Self-Driven Particles

Tamás Vicsek,[1,2] András Czirók,[1] Eshel Ben-Jacob,[3] Inon Cohen,[3] and Ofer Shochet[3]

VOLUME 75, NUMBER 6 PHYSICAL REVIEW LETTERS 7 AUGUST 1995

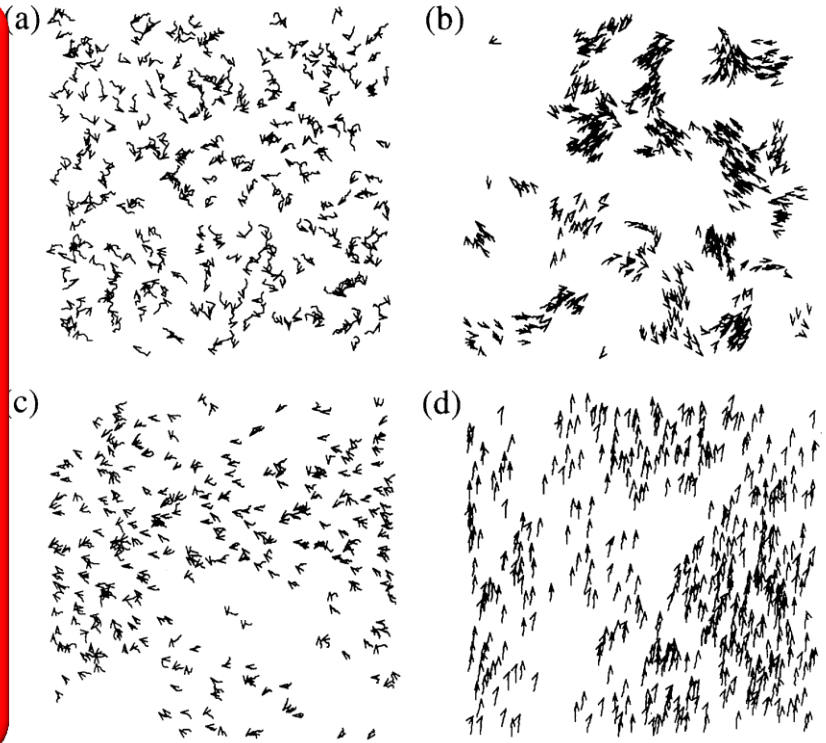
directions θ. the velocities $\{\mathbf{v}_i\}$ of the particles were determined simultaneously at each time step, and the position of the ith particle updated according to

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta t\,. \qquad (1)$$

Here the velocity of a particle $\mathbf{v}_i(t+1)$ was constructed to have an absolute value v and a direction given by the angle $\theta(t+1)$. This angle was obtained from the expression

$$\theta(t+1) = \langle\theta(t)\rangle_r + \Delta\theta\,, \qquad (2)$$

where $\langle\theta(t)\rangle_r$ denotes the average direction of the velocities of particles (including particle i) being within a circle of radius r surrounding the given particle. The average direction was given by the angle $\arctan[\langle\sin(\theta(t))\rangle_r/\langle\cos(\theta(t))\rangle_r]$. In Eq. (2) $\Delta\theta$ is a



N=300 Particles

Modeling of Emergence

Emergent Behavior in Flocks

Felipe Cucker Steve Smale

July 13, 2005

Vicsek's model is motivated by the idea that bird i adjusts its velocity towards the average of its neighbors' velocities. With (our first modification) $\mathbb{R}^2$ replaced by Euclidean space $\mathbb{E}^3$ and the heading θ replaced by the velocity v. The model is

$$\begin{aligned} x_i(t+1) &= x_i(t) + v_i(t) \\ v_i(t+1) &= \frac{1}{n_i(t)} \sum_{j \in \mathcal{N}_i(t)} v_j(t). \end{aligned} \tag{1}$$

where $x_i, v_i \in \mathbb{E}^3$ for $i = 1, \ldots, k$ and time $t = 0, 1, 2 \ldots$. Here $\mathcal{N}_i(t) = \{j \leq k \mid \|x_i(t) - x_j(t)\| \leq r\}$ and $n_i(t) = \#\mathcal{N}_i(t)$ for some $r > 0$.

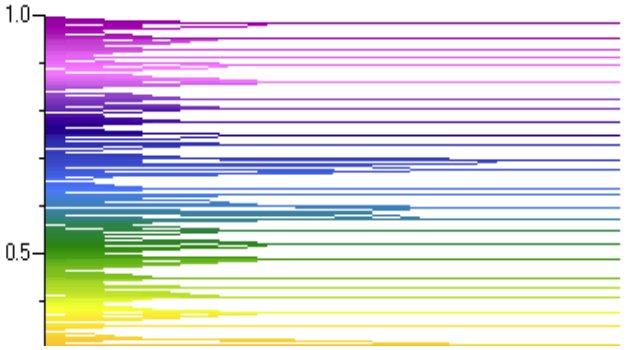
We also consider evolution for continuous time. The corresponding model can be given by the system of differential equations

$$\begin{aligned} x' &= v \\ v' &= -L_x v. \end{aligned} \tag{7}$$

Interesting Computer Simulation of the system (7) ! O.Q

Modeling of Emergence

Hegselmann-Krause-model (HK-model) since 2000



The HK-model is a process of consensus formation with bounded confidence: A systems of points for $\varepsilon > 0$ and $t = 0, 1, 2, \ldots$

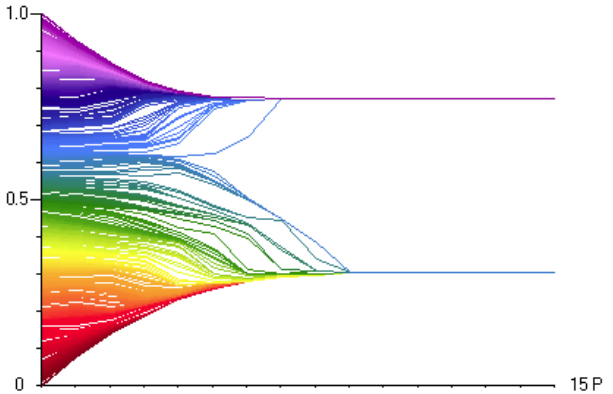
$$x_i(t+1) = \frac{1}{\sharp I(i,t)} \sum_{j \in I(i,t)} x_j(t)$$

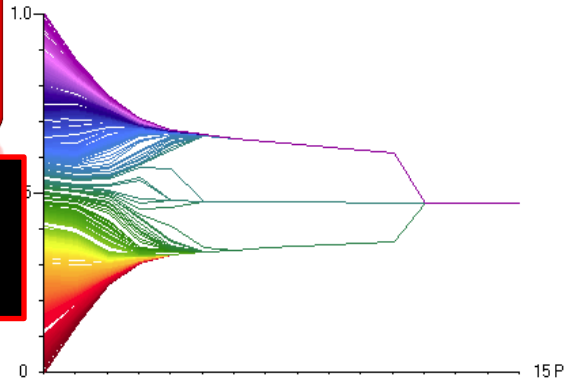
$$I(i,t) = \{j \,|\, |x_i(t) - x_j(t)| \leqslant \varepsilon\}$$

Consensus is the state (c is constant)

$$\lim_{t \to \infty} x_i(t) = c.$$

The HK-model is also known as OPINION DYNAMICS.





Interesting theorem under which conditions consensus is reached ! O.Q

Energy-based-model (Zahri 2009)

Starting from the ε-energy of a position resp. to m (pos. measure):

$$e_\varepsilon(x, m) = \int_{d(x,y)\leqslant\varepsilon} d^2(x, y)m(dy),$$

-extending the HK-model-, we

1. replace the Euclidean space by a metric space (X, d),
2. let the agents move to where the local energy is minimal,
3. Agents do not move simultaneously,

and observe the total ε-energy of m

$$E_\varepsilon(m) = \int_X e_\varepsilon(x, m)m(dx),$$

Finite metric space
Local and global energy map

Definition

(X, d) finite metric space, $m \in M_+(X)$ (nonnegative measures), $\varepsilon > 0$. The ε-energy of m is

$$E_\varepsilon(m) = \sum_{d(x,y)\leqslant\varepsilon} m(x)m(y)d^2(x,y).$$

The $\varepsilon-$ energy of point $a \in X$ with respect to m is

$$e_\varepsilon(a,m) = \sum_{d(a,y)\leqslant\varepsilon} m(y)d^2(a,y).$$

Note: $e_\varepsilon(\cdot, m)$ and $E_\varepsilon(m)$ are resp. local and global energies.

Finite metric space: Moves of a Mass-Point

A pair $(a, a^*) \in X \times X$ operates on the set $M_+(X)$ as

$$m \longrightarrow (a, a^*, m),$$

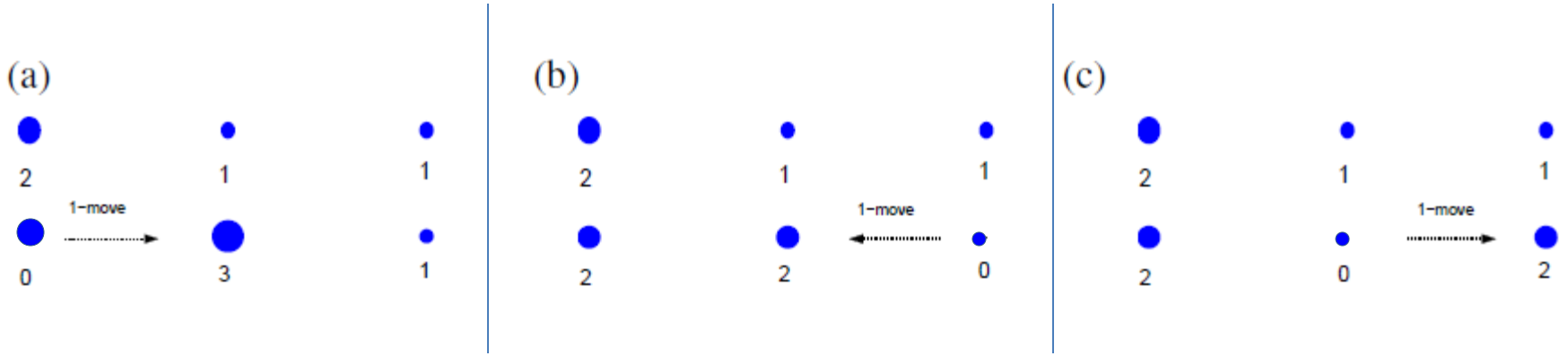
where

$$m^* = (a, a^*, m)(x) = \begin{cases} m(x); & \text{if} \quad x \notin \{a, a^*\}, \\ 0; & \text{if} \quad x = a, \\ m(a) + m(a^*); & \text{if} \quad x = a^*. \end{cases}$$

Note that

- when a moves to a^*, m moves to $m^* = (a, a^*, m)$, and
- $m(X) = m^*(X)$.

Finite metric space: Example of moves



(a) m *is transformed to* m_a^* *with* $E(m) = E(m_a^*) = 6.$

(b) m *is transformed to* m_b^* *with* $E(m) = 6 < E(m_b^*) = 8.$

(c) m *is transformed to* m_c^* *with* $E(m) = 6 > E(m_c^*) = 0.$

We are looking for a local rule for a move of particles satisfying the case (c).

Finite metric space: ε-Move: Definition

A pair (m, m^*) is $\varepsilon-$move, if there is a pair (a, a^*) such that:

(i) $m^* = (a, a^*, m)$,

(ii) $d(a, a^*) \leqslant \varepsilon$,

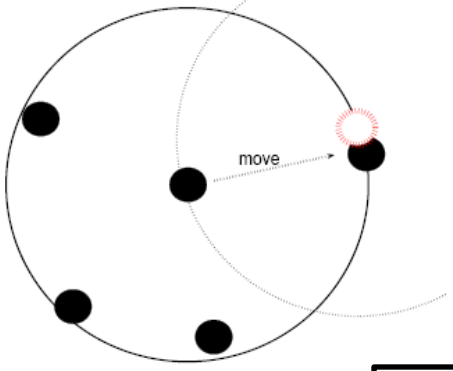
(iii) $e_\varepsilon(a, m) + m(a)d^2(a, a^*) > e_\varepsilon(a^*, m)$.

$m^1, m^2, \ldots$ is $\varepsilon-$condensing if (m^i, m^{i+1}) is $\varepsilon-$move.

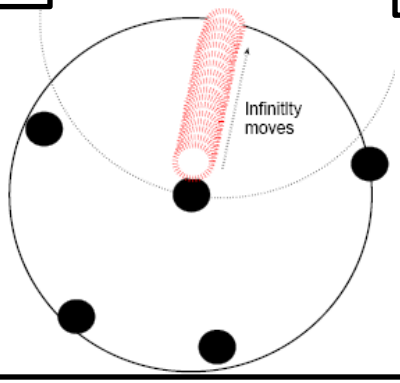
Interpretation

a moves to a^* if $d(a^*, a) \leqslant \varepsilon$ and $e(a^*, m^*) < e(a, m)$.

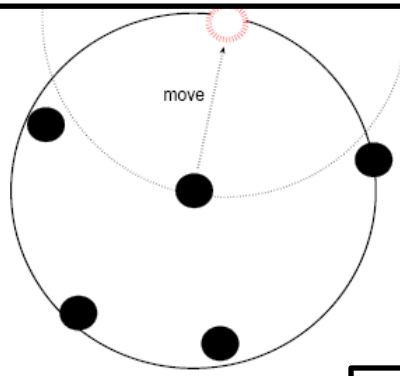
Collision to other particle



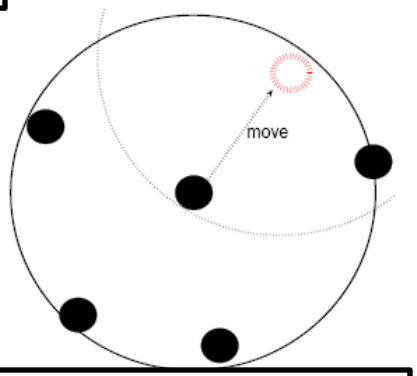
Isolation in infinitely steps



Isolation in one step



Energy minimizing step



Finite metric space: Convergence theorem

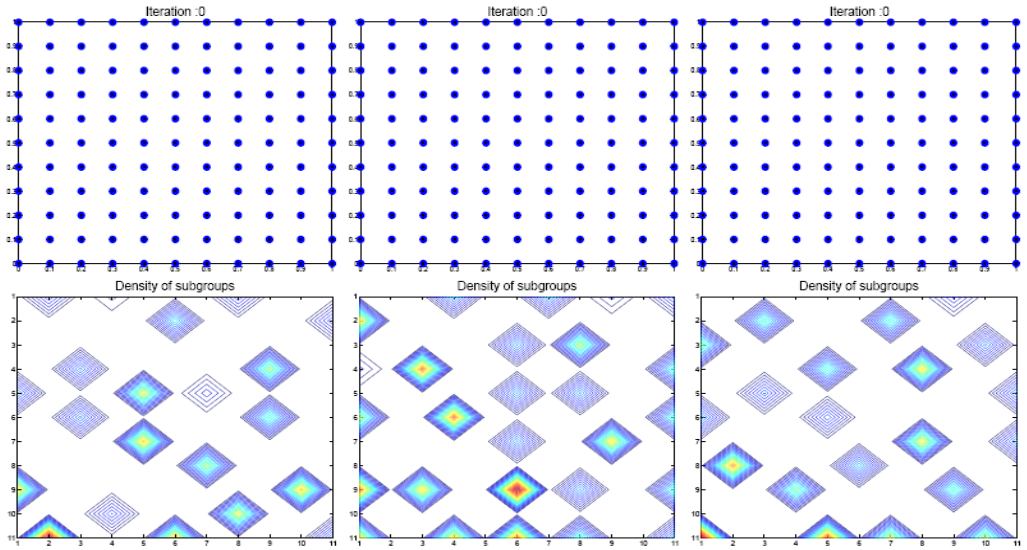
Theorem

A singularly $\varepsilon-$condensing sequence is finite.

Only one move at a time is allowed. Below are three simulation in a finite metric space

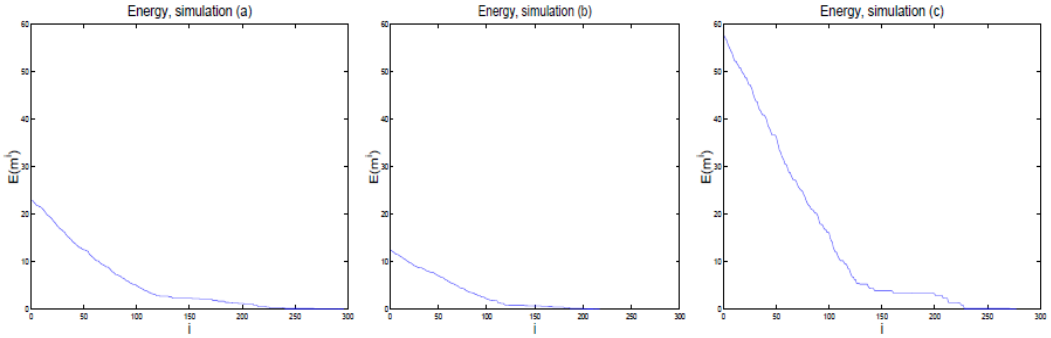
Initial State

Finale State



All energies are decreasing functions in time

Corresponding energies



Continuous metric space

In general E is a not continuous function of m.

Example

For $X = \mathbb{R}$, $S(m) = \{1, 2\}$, $\varepsilon = 1$ and a given m with

$$m = \sum_{x \in \{1,2\}} m(x)\delta_x = \delta_1 + \delta_2 \Rightarrow E(m) = 2.$$

let m_j be a sequence of p.m. defined as $m_j = \delta_1 + \delta_{2+\frac{1}{j}}$, it follows $E(\lim_j m_j) = E(m) = 2 \neq \lim_j E(m_j) = 0$. Hence, the map E is not continuous in m.

Intensity function

Definition

A continuous function φ, which satisfies:

$$\begin{aligned} \varphi &: X \times X \longrightarrow [0,1];\ \varepsilon, \theta > 0; \\ \varphi(x,y) &:= \begin{cases} 1, & \text{if } d(x,y) \leqslant \varepsilon, \\ 0, & \text{if } d(x,y) \geqslant \varepsilon + \theta. \end{cases} \end{aligned}$$

is called **intensity function**.

Example:

$$\begin{aligned} \varphi &: \mathbb{R} \times \mathbb{R} \longrightarrow [0,1];\ \varepsilon, \theta > 0; \\ \varphi(x,y) &:= \begin{cases} 1, & \text{if } |x-y| \leqslant \varepsilon, \\ 0, & \text{if } |x-y| \geqslant \varepsilon + \theta. \\ \left(\frac{\varepsilon+\theta-|x-y|}{\theta}\right) & \text{otherwise.} \end{cases} \end{aligned}$$

Continuous metric space

Definition

The continuous energy function of m is

$$\begin{aligned} E &: M_+(X) \longrightarrow \mathbb{R}^+ \\ E(m) &= \int_X \int_X \varphi(x,y) d^2(x,y) m(dx) m(dy) \\ &= \sum_{x,y} m(x) m(y) \varphi(x,y) d^2(x,y) \end{aligned}$$

The energy of point $a \in X$ with respect to m is

$$e(a,m) = \sum_y m(y) \varphi(a,y) d^2(a,y).$$

where φ is an intensity function.

Continuous metric space

Definition

A pair (m, m^*) is called condensing, if there is $(a, a^*) \in X \times X$ such that

(*i*) $m^* = (a, a^*, m)$ and

(*ii*) a^* minimizing $y \to e(y, m^*)\varphi(a, y)$, with $d(a, y) \leqslant \varepsilon + \theta$.

A sequence of non negative measures $m^1, m^2, \ldots$ is called condensing, if for every i the pair (m^i, m^{i+1}) is condensing. A mapping f such that $m^* = f(m)$ is called condensing.

Effectively condensing

Definition

A singularly condensing sequence $m^1, m^2, \ldots$ is called **effectively condensing sequence** if there exists $c > 0$ such that

(*i*) $m^{i+1} = (a, a^*, m^i)$,

(*ii*) $E(m^i) - E(m^{i+1}) \geqslant c\alpha(m^i)$,

where $\alpha(m^i) = \max_y \{\varphi(y, a)d^2(y, a) | y \in S(m^i)\}$.

From

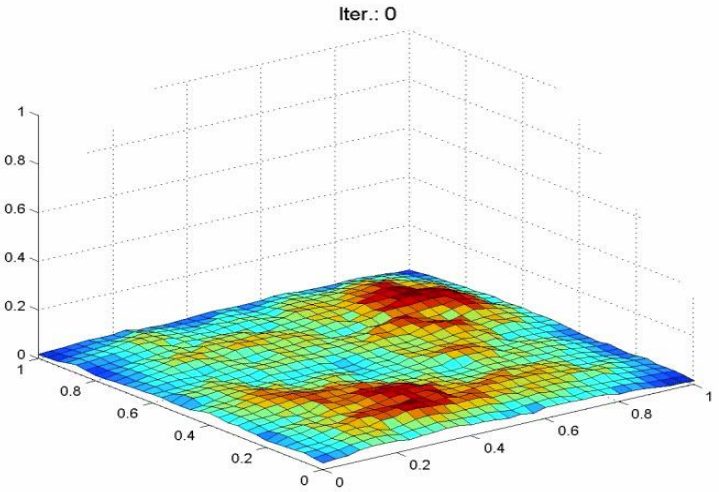
$$\begin{aligned} E(m) - E(m^*) &= 2m(a)\big[e(a, m) - e(a^*, m^*)\big] \text{and} \\ c &= 2\min_a \{m(a) | a \in S(m^i)\}, \end{aligned}$$

follow the existence of the effectively condensing sequence.

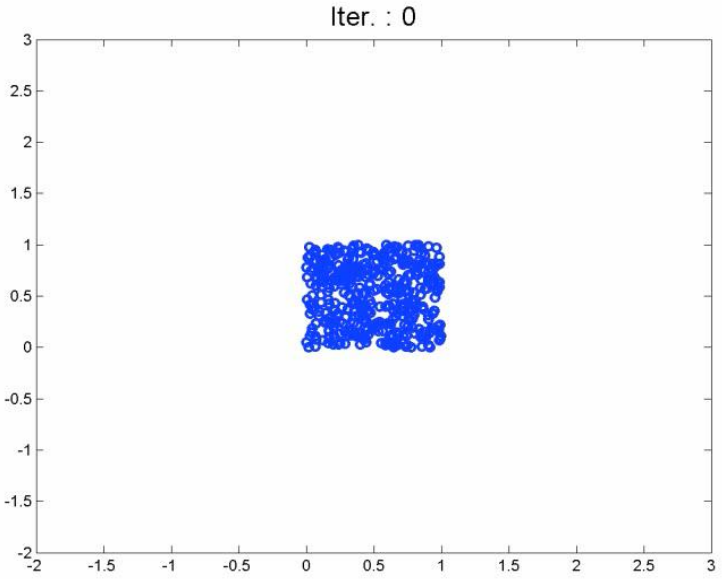
Convergence Theorem & Simulation

Theorem

If (X, d) is a compact metric space, then every effectively condensing sequence $m^1, m^2, \ldots$ is finite.

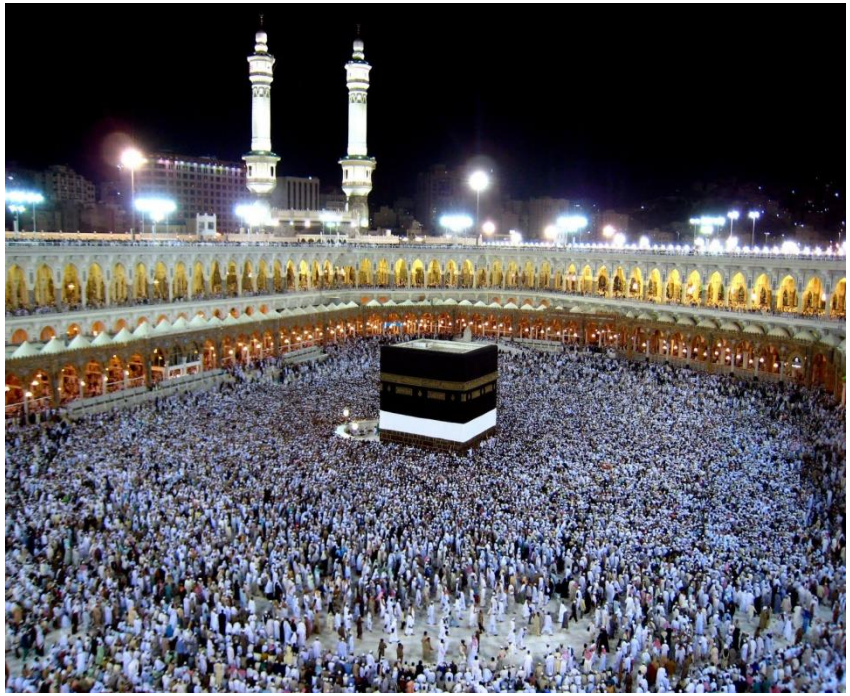


Density function of Condensing particles in a two dimensional continuous metric space

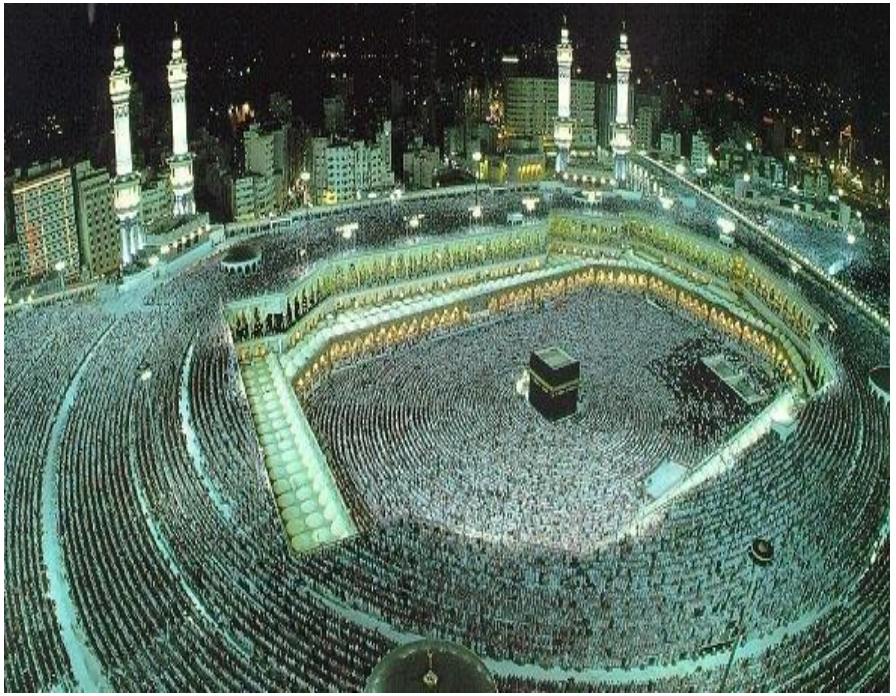


Segregation of particles in a two dimensional continuous metric space

Very Known Self-Organizing Dynamics



The longest Self-Organizing dynamics of all times. (AL-TAWAF)



The largest Self-Organizing Condensing of all times. (AL-SALAT)

Some References:

❖ Mostafa Zahri, **Energy-based model of condensing on metric spaces: Modeling, analysis and simulations of condensing of particles** SVH-Verlag (Sept. 2009) ISBN-10: 3838110811

❖ M. Zahri, **Energy-based model for self-organizing groups,** Proceeding, 1st Hispano-Moroccan Days on Applied Mathematics and Statistics, Vol 1, pp 411-416, 2008

❖ M. Zahri, **Energy-based model of forming subgroups on finite metric space,** Hochschulpublikation, UB Goethe-Universitaet Frankfurt 2009

❖ M. Zahri, **Condensing of self-organizing groups,** Hochschulpublikation, UB Goethe-Universitaet Frankfurt 2010

Thanks