

Collective Intelligence

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Al Madinah 03 Dec. 2010

Motivation

Cells



Bees



Birds



- Self-Organization,
- Emergence,
- Non-linear systems,
- Distributive computing on networks
- **Collective intelligence,**
- **From individual to collective intelligence ?**

MOTIVATION: COLLECTIVE INTELLIGENCE

- **Using the notion of MASSLESS Materiel Photon:**

A. Einstein (about 1900) introduces the revolutionary energy equation **$E=mc^2$** (proved by Max Planck 18 years later)

- Converse case is also interesting **i.e. MATERIELESS Masses!**

(M. Zahri) introduces 2007 the mass-dependent energy

$$\mathbf{E}_\varepsilon(\mathbf{m}) = \int_{\mathbf{X}} \mathbf{e}_\varepsilon(\mathbf{x},\mathbf{m})\mathbf{m}(\mathbf{dx}) = \int_{\mathbf{X}} \int_{\mathbf{d}(\mathbf{x},\mathbf{y})\leq\varepsilon} \mathbf{d}^2(\mathbf{x},\mathbf{y})\mathbf{m}(\mathbf{dy})\mathbf{m}(\mathbf{dx})$$

WHAT is MATERIELESS Masses:

Feeling in general : love, Nervosity …

1. How can be measured so what is the measure m → m(dx)
2. How are two individuals x and y distanced → d(x,y)
3. We have to find corresponding metric space

Novel Type of Phase Transition in a System of Self-Driven Particles

Tamás Vicsek,[1,2] András Czirók,[1] Eshel Ben-Jacob,[3] Inon Cohen,[3] and Ofer Shochet[3]

VOLUME 75, NUMBER 6 PHYSICAL REVIEW LETTERS 7 AUGUST 1995

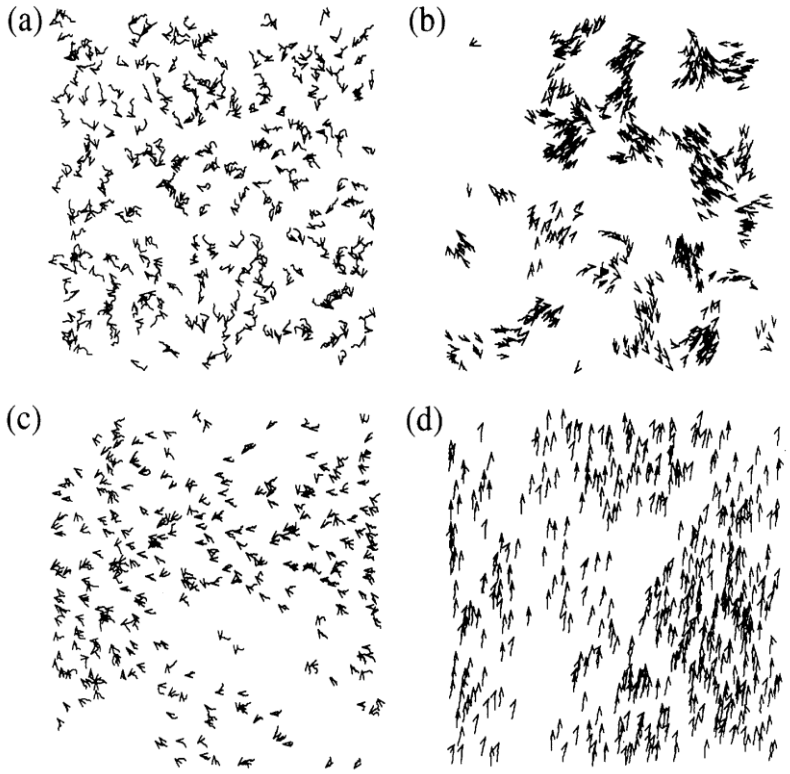
directions θ. the velocities $\{\mathbf{v}_i\}$ of the particles were determined simultaneously at each time step, and the position of the ith particle updated according to

$$\mathbf{x}_i(t+1) = \mathbf{x}_i(t) + \mathbf{v}_i(t)\Delta t\,. \qquad (1)$$

Here the velocity of a particle $\mathbf{v}_i(t+1)$ was constructed to have an absolute value v and a direction given by the angle $\theta(t+1)$. This angle was obtained from the expression

$$\theta(t+1) = \langle\theta(t)\rangle_r + \Delta\theta\,, \qquad (2)$$

where $\langle\theta(t)\rangle_r$ denotes the average direction of the velocities of particles (including particle i) being within a circle of radius r surrounding the given particle. The average direction was given by the angle $\arctan[\langle\sin(\theta(t))\rangle_r/\langle\cos(\theta(t))\rangle_r]$. In Eq. (2) $\Delta\theta$ is a



N=300 Particles

Emergent Behavior in Flocks

Felipe Cucker Steve Smale

July 13, 2005

Vicsek's model is motivated by the idea that bird i adjusts its velocity towards the average of its neighbors' velocities. With (our first modification) $\mathbb{R}^2$ replaced by Euclidean space $\mathbb{E}^3$ and the heading θ replaced by the velocity v. The model is

$$\begin{aligned} x_i(t+1) &= x_i(t) + v_i(t) \\ v_i(t+1) &= \frac{1}{n_i(t)} \sum_{j \in \mathcal{N}_i(t)} v_j(t). \end{aligned} \tag{1}$$

where $x_i, v_i \in \mathbb{E}^3$ for $i = 1, \ldots, k$ and time $t = 0, 1, 2 \ldots$. Here $\mathcal{N}_i(t) = \{j \leq k \mid \|x_i(t) - x_j(t)\| \leq r\}$ and $n_i(t) = \#\mathcal{N}_i(t)$ for some $r > 0$.

We also consider evolution for continuous time. The corresponding model can be given by the system of differential equations

$$\begin{aligned} x' &= v \\ v' &= -L_x v. \end{aligned} \tag{7}$$

Moreover Smale and Cucker (2007) define the concept

Mathematic of Emergence

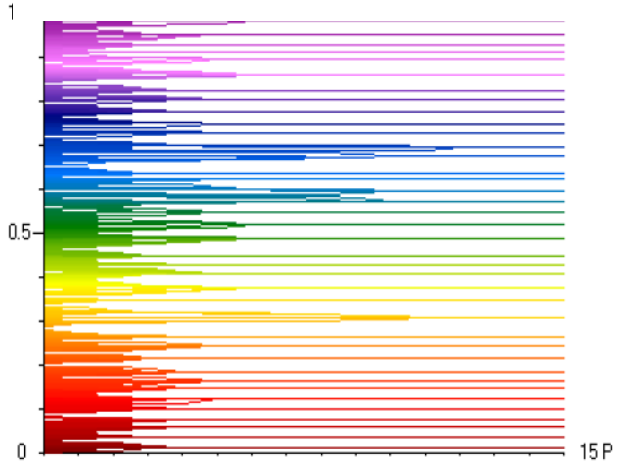
OPINION DYNAMICS AND BOUNDED CONFIDENCE MODELS, ANALYSIS, AND SIMULATION*

Rainer Hegselmann Ulrich Krause

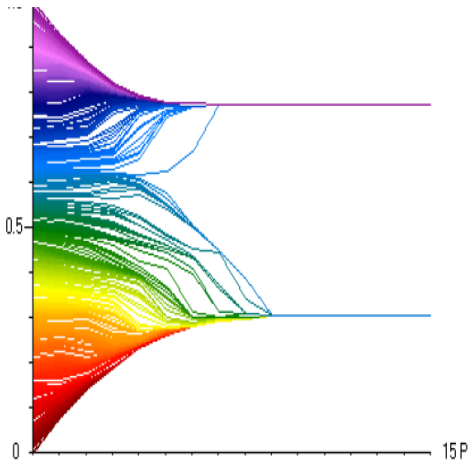
Journal of Artifical Societies and Social Simulation (JASSS) vol.5, no. 3, 2002

Abstract: When does opinion formation within an interacting group lead to consensus, polarization or fragmentation? The article investigates various models for the dynamics of continuous opinions by analytical methods as well as by computer simulations.

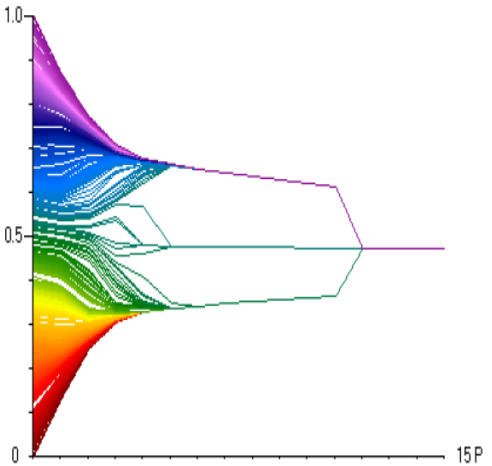
$$x_i(t+1) = \left|I(i,x(t))\right|^{-1} \sum_{j \in I(i,x(t))} x_j(t) \text{ for } t \in T, \quad \text{where } I(i,x) = \left\{1 \le j \le n \,\middle|\, \left|x_i - x_j\right| \le \varepsilon_i\right\}$$



(a) $\varepsilon_l = \varepsilon_r = 0.01$



(b) $\varepsilon_l = \varepsilon_r = 0.15$



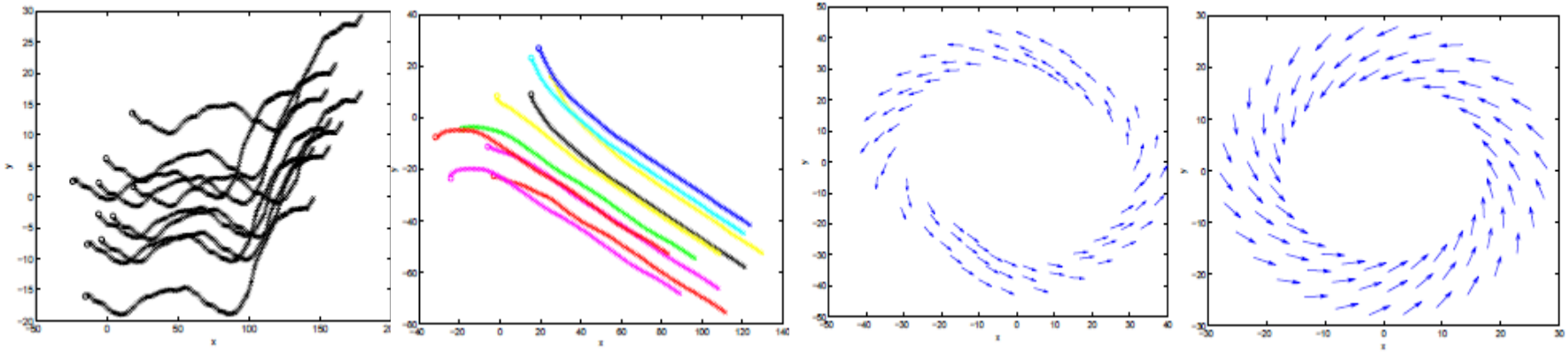
(c) $\varepsilon_l = \varepsilon_r = 0.25$

Discrete and Continuous Models of the Dynamics of Pelagic Fish:

Alethea B. T. Barbaro * Björn Birnir † June 10, 2007
Kirk Taylor

Birnir's ODEs: The position of the fish is expressed in polar coordinates r and θ

$$\begin{cases} \dot{r}_k = v_k \cos(\phi_k - \theta_k) \\ r_k \dot{\theta}_k = v_k \sin(\phi_k - \theta_k) \\ \dot{v}_k = \frac{\alpha}{N^2} \sum_{j=1}^{N} v_j \sum_{j=1}^{N} \cos(\phi_j - \phi_k) - \alpha v_k \\ v_k \dot{\phi}_k = \frac{\alpha}{N^2} \sum_{j=1}^{N} v_j \sum_{j=1}^{N} \sin(\phi_j - \phi_k) \end{cases}$$



Abstract: In this paper, we study simulations of the schooling and swarming behavior of a mathematical model for the motion of pelagic fish. We use a derivative of a discrete model of interacting particles originated by Vicsek et al. …

G u t a c h t e n

über die Dissertationsschrift

„Condensing on Metric Spaces. Modeling, Analysis, and Simulation“

vorgelegt von

Dipl.Math. / B.Sc.Stat. Mostafa Zahri

In dieser Arbeit entwickelt und untersucht Herr Zahri eine vielversprechende Erweiterung des sogenannten HK-Modells [15, 16, 17, 19, 20]. Bei letzterem handelt es sich um ein nichtlineares dynamisches System in diskreter Zeit, gegeben für $i = 1, 2, \ldots, n$ und $t = 0, 1, 2, \ldots$ durch

$$x_i(t+1) = \frac{1}{\sharp I(i,t)} \sum_{j \in I(i,t)} x_j(t), \quad x_i(0) \in [0,1].$$

Related and present works

1. Vicsek and al., Flocking, modeling and simulation (1995).
2. Hegselmann and Krause, Consensus, Opinion dynamics (2002).
3. Smale and Cucker, Flocking of birds, Math. of emergence (2006).
4. Birnir, Schooling of fishes (2007).

Present work:

1. Energy based model as extension of the HK-model.
2. From local interactions to global effect.
3. Analysis of the energies in a group.

FMS: Local and global energy map

Definition

(X, d) finite metric space, $m \in M_+(X)$ (nonnegative measures), $\varepsilon > 0$. The ε-energy of m is

$$E_\varepsilon(m) = \sum_{d(x,y) \leqslant \varepsilon} m(x)m(y)d^2(x,y).$$

The $\varepsilon-$ energy of point $a \in X$ with respect to m is

$$e_\varepsilon(a,m) = \sum_{d(a,y) \leqslant \varepsilon} m(y)d^2(a,y).$$

Note: $e_\varepsilon(\cdot, m)$ and $E_\varepsilon(m)$ are resp. local and global energies.

FMS: Moves of Point Mass

A pair $(a, a^*) \in X \times X$ operates on the set $M_+(X)$ as

$$m \to (a, a^*, m),$$

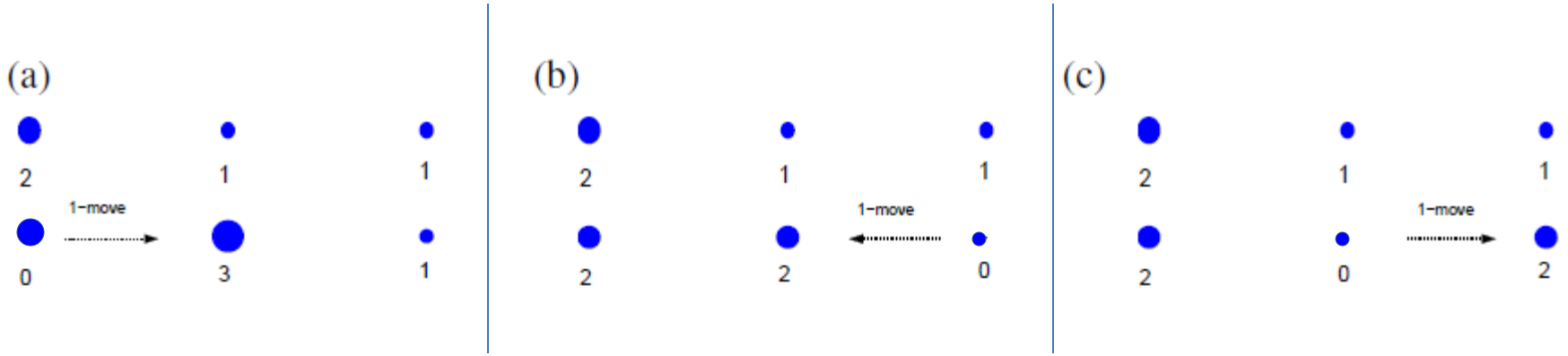
where

$$m^* = (a, a^*, m)(x) = \begin{cases} m(x); & \text{if} \quad x \notin \{a, a^*\}, \\ 0; & \text{if} \quad x = a, \\ m(a) + m(a^*); & \text{if} \quad x = a^*. \end{cases}$$

Note that

- when a moves to a^*, m moves to $m^* = (a, a^*, m)$, and
- $m(X) = m^*(X)$.

FMS: Example of moves



(a) m *is transformed to* m_a^* *with* $E(m) = E(m_a^*) = 6.$

(b) m *is transformed to* m_b^* *with* $E(m) = 6 < E(m_b^*) = 8.$

(c) m *is transformed to* m_c^* *with* $E(m) = 6 > E(m_c^*) = 0.$

We are looking for a local rule for a move of particles satisfying the case (c).

FMS: $\mathcal{E}$-Move: Definition

A pair (m, m^*) is $\varepsilon-$move, if there is a pair (a, a^*) such that:

$$
\begin{aligned}
&(i) \quad m^* = (a, a^*, m),\\
&(ii) \quad d(a, a^*) \leqslant \varepsilon,\\
&(iii) \quad e_\varepsilon(a, m) > e_\varepsilon(a^*, m^*).
\end{aligned}
$$

$m^1, m^2, \ldots$ is $\varepsilon-$condensing if (m^i, m^{i+1}) is $\varepsilon-$move.

Interpretation

Each agent *a* of the collection *m* is permanently looking for positions minimizing its local energy

FMS: Convergence theorem

Theorem

A singularly (one move at a time) $\varepsilon-$condensing sequence is finite. (in other words: The sequence converges.)

The proof is a consequence of the following lemmas.

Lemma

Let $m \in M_+(X), a, a^* \in X$ such that $d(a, a^*) \leqslant \varepsilon$. Then

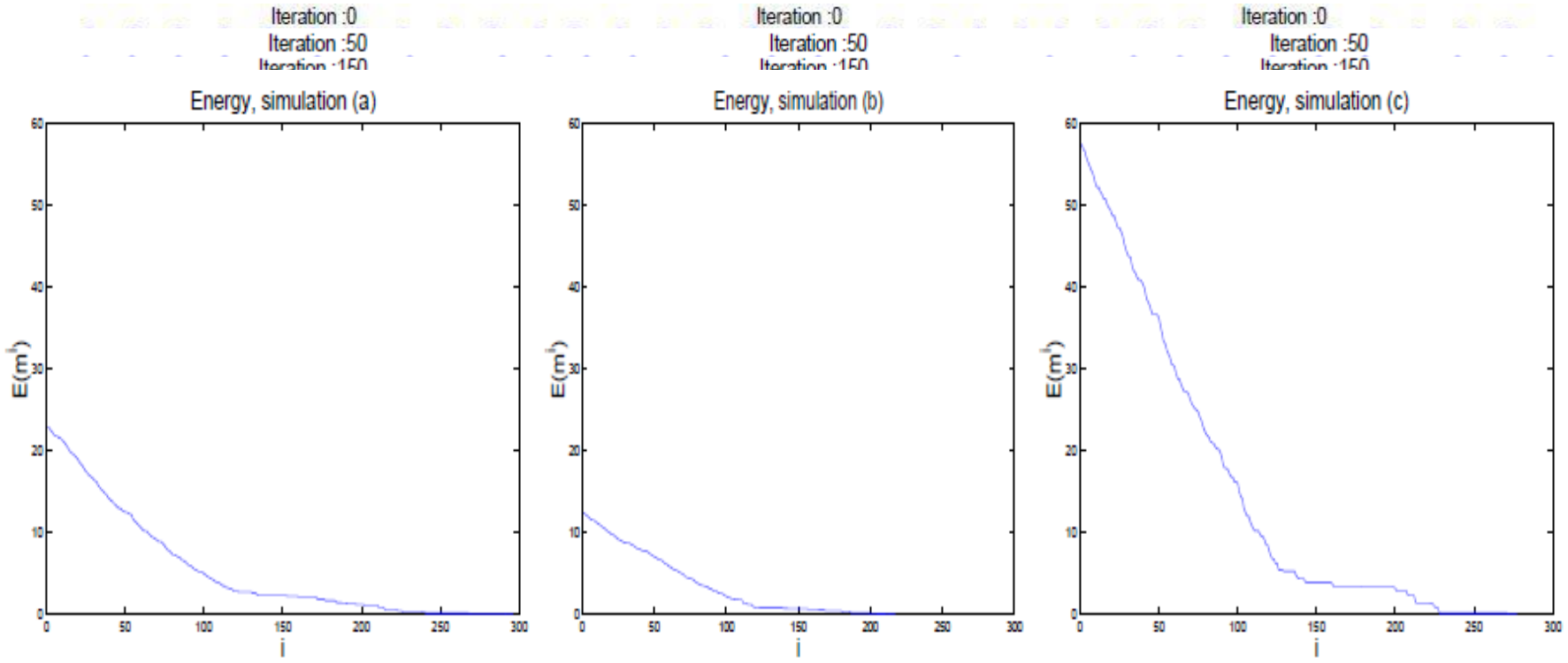
$$E_\varepsilon(m) - E_\varepsilon(m^*) = 2m(a)\left[e_\varepsilon(a, m) - e_\varepsilon(a^*, m) + m(a)d^2(a, a^*)\right],$$

Lemma

For $m \in M_+(X)$ let $n(m)$ be the number of elements $a \in X$ such that $m(a) > 0$. If $m^1, m^2, \ldots$ is a sequence of measures on X which is singularly and $\varepsilon-$condensing, then

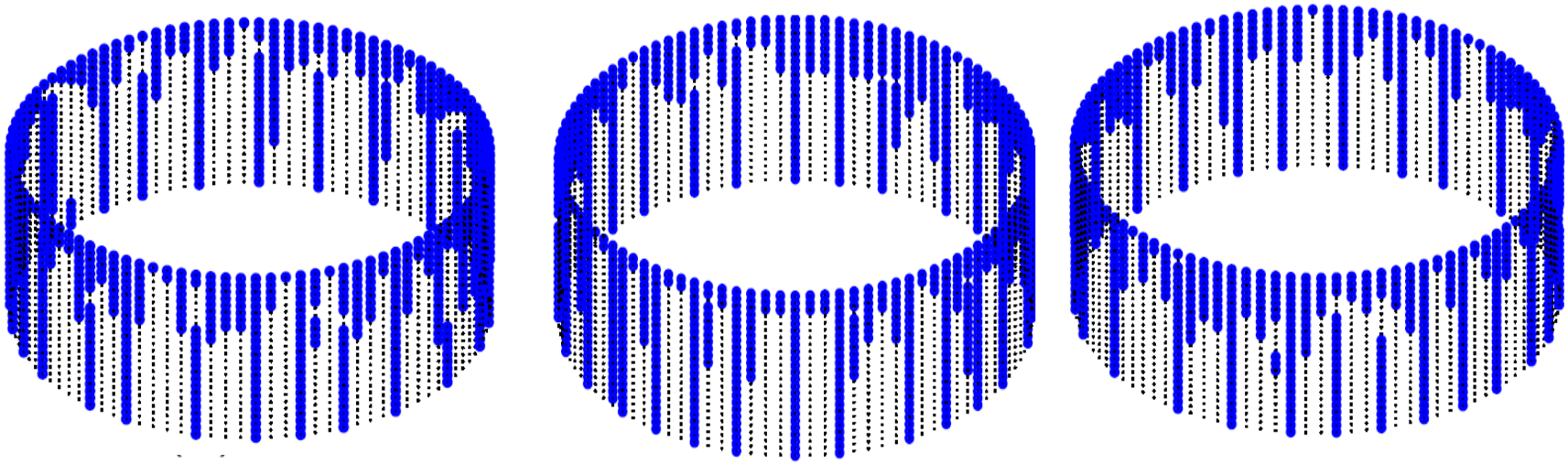
1. *$i \longrightarrow E_\varepsilon(m^i)$ is strictly decreasing*
2. *$i \longrightarrow n(m^i)$ is decreasing*

FMS: Simulation, Euclidean



Results of simulations in an (Euclidean) finite metric space.

FMS: Simulation, Circular



Results of simulations in a finite circular metric space.

CMS: Energy function

In general E is a not continuous function of m.

Example

For $X = \mathbb{R}$, $S(m) = \{1, 2\}$, $\varepsilon = 1$ and a given m with

$$m = \sum_{x \in \{1,2\}} m(x)\delta_x = \delta_1 + \delta_2 \Rightarrow E(m) = 2.$$

let m_j be a sequence of p.m. defined as $m_j = \delta_1 + \delta_{2+\frac{1}{j}}$, it follows $E(\lim_j m_j) = E(m) = 2 \neq \lim_j E(m_j) = 0$. Hence, the map E is not continuous in m.

CMS: Intensity function, Example

Definition

A continuous function φ, which satisfies:

$$\begin{aligned} \varphi &: X \times X \longrightarrow [0,1];\ \varepsilon, \theta > 0; \\ \varphi(x,y) &:= \begin{cases} 1, & \text{if } d(x,y) \leqslant \varepsilon, \\ 0, & \text{if } d(x,y) \geqslant \varepsilon + \theta. \end{cases} \end{aligned}$$

is called **intensity function**.

Example:

$$\begin{aligned} \varphi &: \mathbb{R} \times \mathbb{R} \longrightarrow [0,1];\ \varepsilon, \theta > 0; \\ \varphi(x,y) &:= \begin{cases} 1, & \text{if } |x-y| \leqslant \varepsilon, \\ 0, & \text{if } |x-y| \geqslant \varepsilon + \theta. \\ \left(\frac{\varepsilon+\theta-|x-y|}{\theta}\right) & \text{otherwise.} \end{cases} \end{aligned}$$

CMS: Continuous energy function

Definition

The continuous energy function of m is

$$\begin{aligned} E &: M_+(X) \longrightarrow \mathbb{R}^+ \\ E(m) &= \int_X \int_X \varphi(x,y) d^2(x,y) m(dx) m(dy) \\ &= \sum_{x,y} m(x) m(y) \varphi(x,y) d^2(x,y) \end{aligned}$$

The energy of point $a \in X$ with respect to m is

$$e(a,m) = \sum_y m(y) \varphi(a,y) d^2(a,y).$$

where φ is an intensity function.

CMS: Condensing sequence

Definition

A pair (m, m^*) is called condensing, if there is $(a, a^*) \in X \times X$ such that

(*i*) $m^* = (a, a^*, m)$ and

(*ii*) a^* minimizing $y \to e(y, m^*)\varphi(a, y)$, with $d(a, y) \leqslant \varepsilon + \theta$.

A sequence of non negative measures $m^1, m^2, \ldots$ is called condensing, if for every i the pair (m^i, m^{i+1}) is condensing. A mapping f such that $m^* = f(m)$ is called condensing.

CMS: Effectively condensing

Definition

A singularly condensing sequence $m^1, m^2, \ldots$ is called **effectively condensing sequence** if there exists $c > 0$ such that

(*i*) $m^{i+1} = (a, a^*, m^i)$,

(*ii*) $E(m^i) - E(m^{i+1}) \geqslant c\alpha(m^i)$,

where $\alpha(m^i) = \max_y \{\varphi(y, a)d^2(y, a) | y \in S(m^i)\}$.

From

$$\begin{aligned} E(m) - E(m^*) &= 2m(a)\left[e(a, m) - e(a^*, m^*)\right] \text{and} \\ c &= 2\min_a \{m(a) | a \in S(m^i)\}, \end{aligned}$$

follow the existence of the effectively condensing sequence.

CMS: Convergence theorem

Theorem

If (X, d) is a compact metric space, then every effectively condensing sequence $m^1, m^2, \ldots$ is finite.

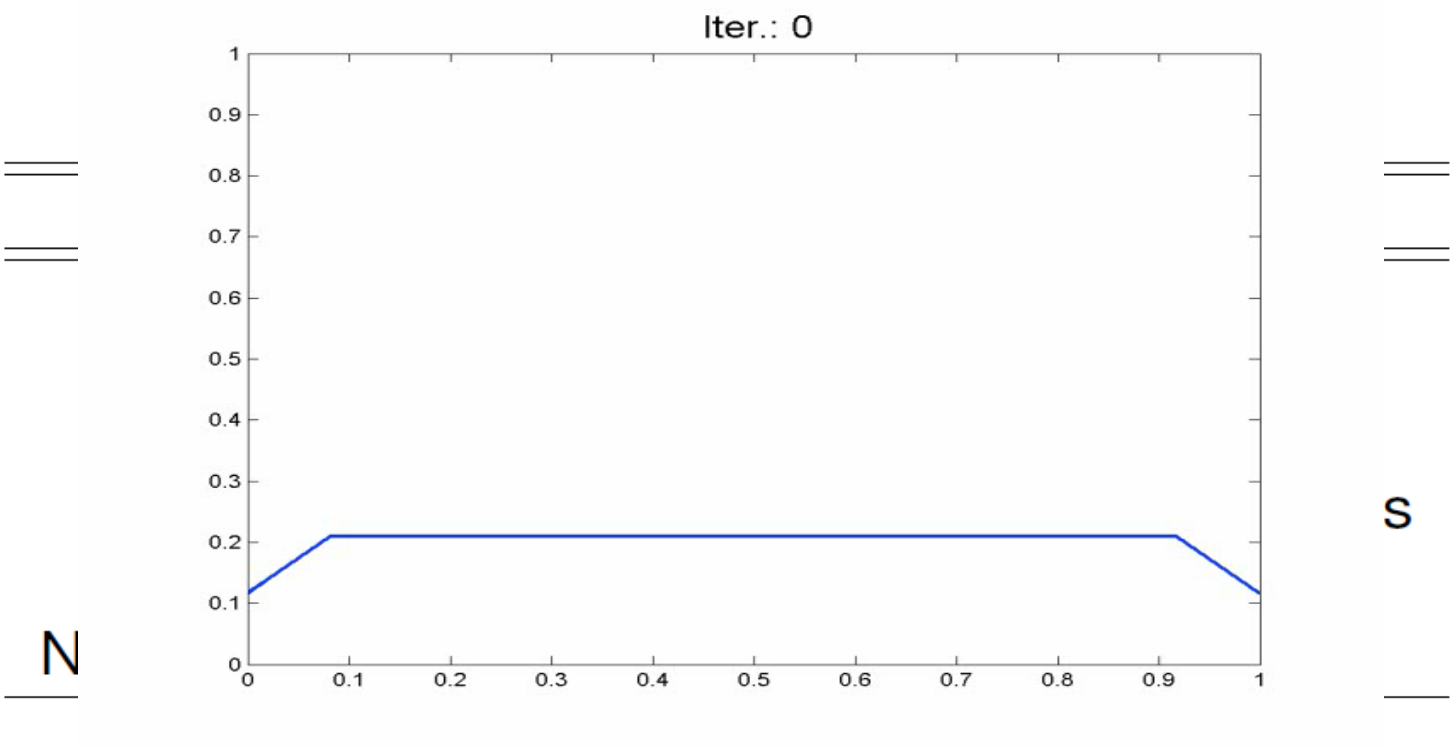
Proof idea: Since X is compact and

Lemma

Suppose that $m^1, m^2, \ldots$ is an effectively condensing.

$$\text{if } \lim_{i \to \infty} E(m^i) = E(m), \text{ then } \alpha(m) = 0.$$

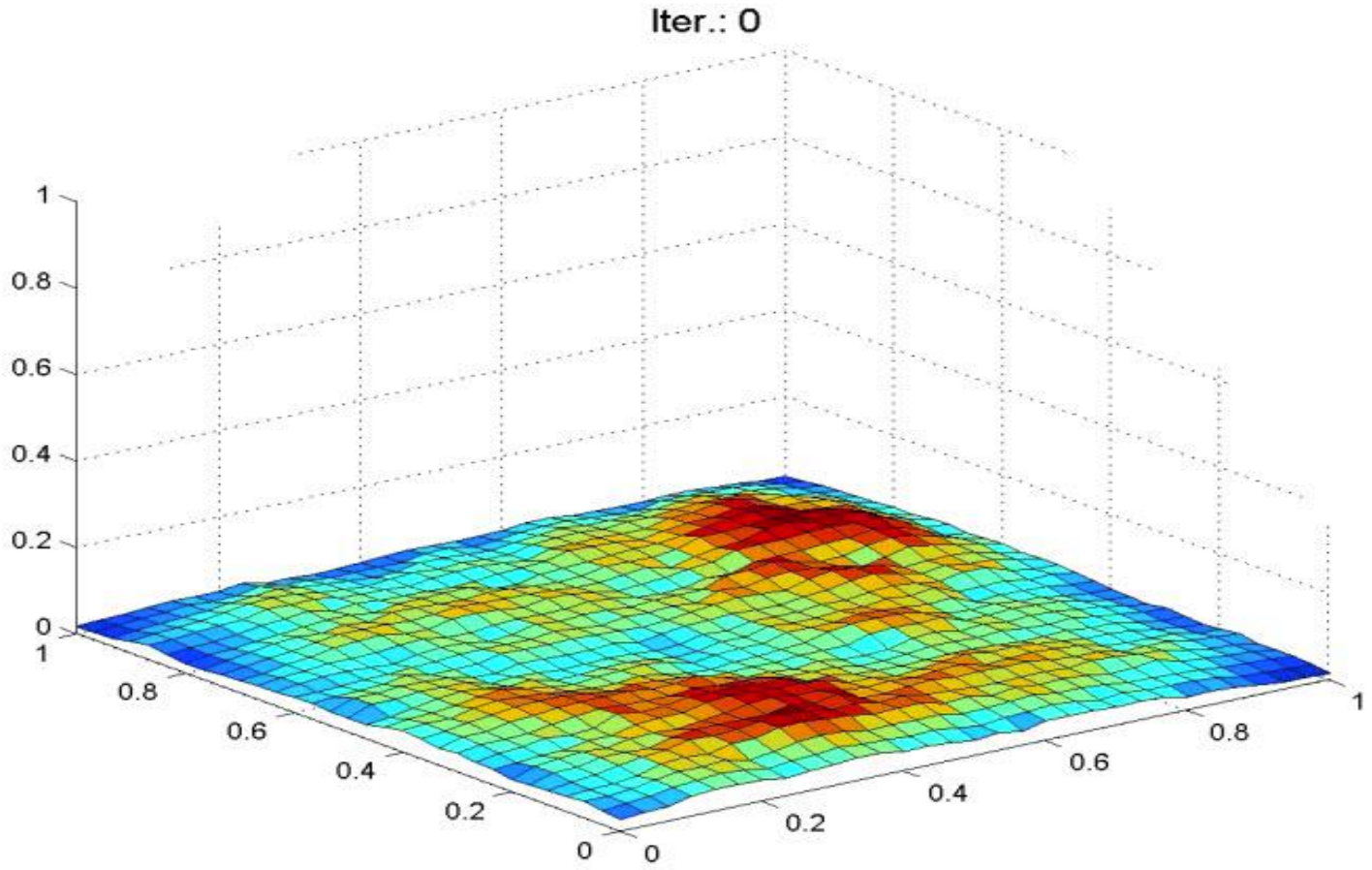
CMS: Simulation, real Line



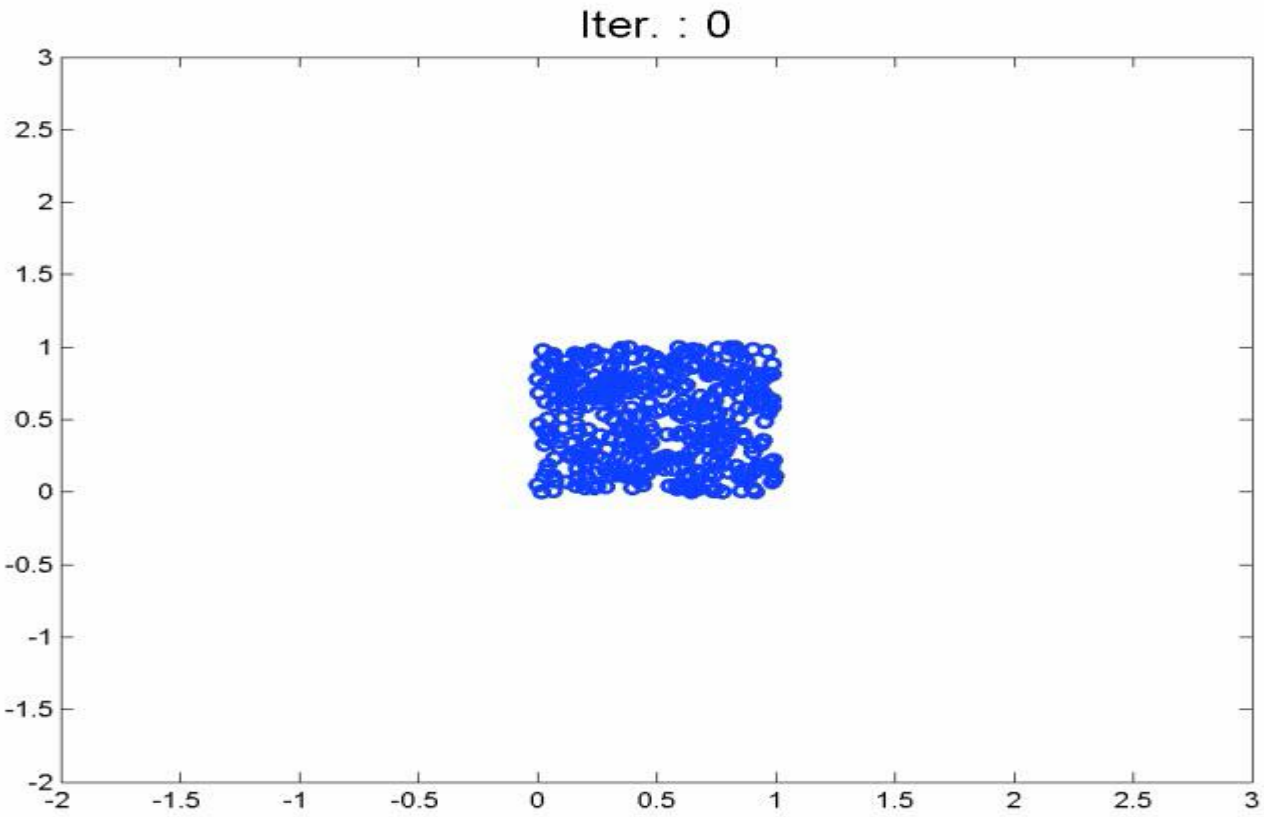
Results of simulations in the real line.

CMS: Simulation, real plane

Two dimensional density function of a condensing sequence of measures



CMS: Simulation, Segregation



Prohibition of collisions leads to segregation

Danke